Parameter Uncertainty and Non-Linear Monetary Policy Rules

Peter Tillmann[1]
Justus Liebig University Giessen

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Abstract: Empirical evidence suggests that the instrument rule describing the interest rate setting behavior of the Federal Reserve is non-linear. This paper shows that optimal monetary policy under parameter uncertainty can motivate this pattern. If the central bank is uncertain about the slope of the Phillips curve and follows a min-max strategy to formulate policy, the interest rate reacts more strongly to inflation when inflation is further away from target. The reason is that the worst case the central bank takes into account is endogenous and depends on the inflation rate and the output gap. As inflation increases, the worst-case perception of the Phillips curve slope becomes larger, thus requiring a stronger interest rate adjustment. Empirical evidence supports this form of non-linearity for post-1982 U.S. data.

Keywords: parameter uncertainty, robust control, non-linear Taylor rule, optimal monetary policy, Federal Reserve policy

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[1]Contact: Department of Economics, Justus Liebig University Giessen, Licher Str. 62, D-35394 Giessen, E-mail: `peter.tillmann@wirtschaft.uni-giessen.de`

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1 Introduction

The interest rate setting behavior of central banks is routinely described by estimated interest rate rules. In the baseline specification going back to Taylor (1993), for example, the policy instrument, i.e. the short-term interest rate, is linearly related to contemporaneous inflation and the output gap. These estimated rules perform remarkably well in replicating post-1982 Federal Reserve policy.[2] Moreover, these rules are essential to central bank communication and model building alike.

Recent empirical evidence points to important non-linearities in interest rate setting that are neglected in the standard specification of estimated Taylor rules. For example, Dolado et al. (2004, 2005) include an interaction term between inflation and the output gap in an otherwise standard Taylor rule. They are able to show that policy behaves non-linearly after 1983. Kim et al. (2005) use a flexible nonparametric method to document non-linearity in the Fed's policy rule prior to 1979, but fail to show non-linearity thereafter. Further evidence that Federal Reserve policy becomes more aggressive the further inflation is away from target is provided by Mizen, Kim, and Thanaset (2005) using quantile regression. They estimate the Taylor rule response coefficient at various points of the conditional distribution corresponding to different levels of interest rates.

In addition, central banks frequently announce a target range around their inflation target, i.e. small deviations of inflation from target are tolerated while large deviations are fought vigorously. Federal Reserve officials often refer to current inflation being in a "comfort zone", i.e. a range in which no immediate monetary reaction is required.[3]

A recent literature aims at motivating this apparent non-linearity in interest rate setting. Three approaches can be distinguished, each of which will be portrayed in more detail in the next section. First, the Phillips curve trade-off could be non-linear. This non-linearity would translate into optimal monetary policy. Second, the central bank preferences could be asymmetric and, hence, deviate from the standard linear-quadratic framework. A third way to motivate non-linearity, put forward by Meyer et al. (2001), is to assume that the central banker faces uncertainty about the model describing the economy which is represented by a non-Gaussian distribution.

This paper contributes the analysis of optimal monetary policy under uncertainty. In contrast to some of the papers mentioned before, we do not introduce non-linearity in one of the model elements and then solve for optimal policy that, not surprisingly, eventually also exhibits non-linearity. Rather, this paper presents a mechanism that

[2] See, among others, Clarida et al. (1998, 2000), Judd and Rudebusch (1998), and Jondeau et al. (2004).

[3] See Mishkin (2008a) for a discussion.

generates non-linearity endogenously. To the extent the central banker is concerned about model misspecifications, the resulting policy rule is non-linear. The model nests the standard linear Taylor rule as a special case in the absence of uncertainty. We assume that the central bank is uncertain about a key parameter governing the transmission process of monetary policy, which is the slope of the Phillips curve in an otherwise standard New-Keynesian model. In this paper, the linearity of the Phillips curve and the quadratic nature of the loss function are retained. The key contribution is to show that non-linearity results from optimal monetary policy if the central bank follows a min-max strategy to take account of parameter uncertainty. Policymakers aim at setting interest rates optimally given a particular reference model but, at the same time, admit that they cannot be completely certain about the true model specification. As a result, central banks want to formulate robust policies that are to some extent immune with respect to model disturbances. They set interest rates so as to minimize the maximum harm to the economy.

Given this policy approach, the resulting optimal interest rate rule includes not only the inflation rate and the output gap, but also an interaction term between output and the squared inflation rate. If the central bank is uncertain about the slope of the Phillips curve and follows a worst-case strategy to formulate policy, the interest rates react more strongly to inflation when inflation is further away from target. The reason is that the worst case the central bank takes into account is endogenous and depends on the size of the inflation rate. When inflation is high, the loss from a misspecified parameter is particularly high. Hence, the central banks becomes more vigorous in fighting inflation. A robustness-concerned central bank tolerates small deviations of inflation from target, but strongly counteracts larger movements of inflation. We provide empirical evidence that supports this form of non-linearity for post-1982 U.S. data.

This paper is organized as follows. Section two surveys the literature on the rationale for non-linear monetary policy rules. Section three presents the model and solves for optimal min-max policy under uncertainty. Section four studies the properties of the resulting non-linear instrument rule, while section five provides empirical support for the form of non-linearity analyzed here. Finally, section six draws some conclusions.

2 Non-linear policy rules

As explained in the introduction, non-linearity is a pervasive characteristic of the interest rate setting behavior of many central banks. From a theoretical point of view, non-linearity in the policy rule can be motivated in at least three different ways.

First, the underlying aggregate supply schedule might be non-linear. Nobay and Peel (2000) and Dolado et al. (2005), among others, introduce convexity or concavity in a short-run Phillips curve that nests the linear trade-off as a special case. Eventually, this non-linearity translates into optimal policy leading to a non-linear adjustment of the policy rate

Second, the preferences of the policy maker might not be quadratic in output and inflation. Think of a central bank that puts different weights to positive and negative deviations of output from target or to negative versus positive inflation deviations. These departures from the standard linear-quadratic paradigm ultimately drive non-linear interest rate dynamics. Surico (2007a,b), among others, models asymmetric preferences in a standard New-Keynesian model. The resulting non-linear interest rate rule performs well in the pre-Volcker period but shows fewer signs of asymmetry in the post-Volcker era. Similar models with asymmetric preferences of the policy maker are presented by Ruge-Murcia (2003), Nobay and Peel (2003), and Cukierman and Muscatelli (2008). A closely related literature proposes an opportunistic approach to monetary policy, see Orphanides and Wilcox (2002). According to this view, the Fed tolerates moderate levels of inflation above the target and waits for favorable circumstances to reduce inflation. The result will also be a non-linear interest rate adjustment.

Third, policymakers might face uncertainty. Meyer et al. (2001) and Swanson (2006) show that non-linearities stem from uncertainty about the natural rate of unemployment, formalized by a non-Gaussian prior distribution and a non-linear updating rule. As a result of the signal extraction problem, the central bank is more cautious about adjusting interest rates in response to small output gaps than in a standard Taylor rule but more aggressive when they reach a certain threshold.[4]

This paper adds to the analysis of the third source of non-linearity, i.e. to monetary policy under uncertainty. Policymakers aim at setting interest rates optimally given a particular reference model but, at the same time, admit that they cannot be completely certain about the true model specification. As a result, central banks want to formulate robust policies that are to some extent immune with respect to model disturbances. In contrast to Meyer et al. (2001), the central bank in this paper is unable to entertain a prior distribution over competing parameter realizations. Instead, policy follows a min-max approach. Such a policy concept is also known as a robust control approach to policymaking and was pioneered by Hansen and Sargent (2008).[5] The central bank

[4] A series of speeches by Federal Reserve Governor Meyer provides narrative evidence for this kind of non-linearity, see Meyer (2000).

[5] The special attention policymakers pay to the worst-case outcome is supported by narrative evidence, see Greenspan (2004) or recently Mishkin (2008b).

has a reference model at hand that provides the most likely description of the economy. Under robust control, however, the policymaker believes the model to be misspecified to a certain degree and formulates a policy that is optimal, i.e. that minimizes the central bank's loss function, and at the same time takes the worst-case misspecification into account.[6] This paper shows that, to the extent the central bank is uncertain about a key parameter, the resulting min-max policy rule exhibits an important non-linear element.

This paper uses a minmax approach to address parameter uncertainty. An alternative approach to model monetary policy under uncertainty allows the central bank to be able to attach priors to alternative parameter values. As Adam (2004) argues, min-max decision theory represents the choice of a particular objective function such that Bayesian decisions are insensitive to alternative priors. The choice of the robust control approach is motivated by recent narrative evidence. When he was FOMC member, Frederick Mishkin (2008b) argued that "the design of monetary policy ought to reflect the public's preferences, especially with respect to avoiding particularly adverse economic outcomes". Put differently, he supports the notion that policymakers pay special attention to the worst-case outcome.

As mentioned before, Hansen and Sargent (2008) provide a seminal analysis of robust control problems in economics. Onatski and Williams (2003) use their framework, but offer a more structural analysis of model uncertainty than Hansen and Sargent and apply min-max policy rules to a small empirical model of the U.S. economy. Leitemo and Söderström (2008) apply robust control techniques to a standard New Keynesian model and derive optimal monetary policy. As in Leitemo and Söderström (2008), the model in this paper is simple enough to facilitate an analytical solution of the policy problem. However, in contrast to their contribution, the central bank is uncertain about a particular parameter of the model with the model distortion directly affecting a particular parameter value instead of affecting the disturbance terms. In this sense, the model draws on the work of Onatski and Williams (2003).

3 Optimal policy rules under uncertainty

This section outlines the role of parameter uncertainty and robust monetary policy in an otherwise standard New-Keynesian model.

[6] See Giannoni (2002), Rudebusch (2001), and Söderström (2002) for a more general analysis of monetary policy rules under parameter uncertainty.

3.1 The model

We employ the standard New Keynesian model as a laboratory, see e.g. Woodford (2003) for a complete derivation. The forward-looking Phillips curve (1) and the IS curve (2) represent log-linearised equilibrium conditions of a simple sticky-price general equilibrium model

$$\pi_t = \beta E_t \pi_{t+1} + \kappa_t x_t + \xi_t \tag{1}$$

$$x_t = E_t x_{t+1} - \sigma^{-1} \left(i_t - E_t \pi_{t+1} - r_t^n \right) \tag{2}$$

where π_t is the inflation rate, x_t the output gap, i_t the risk-free nominal interest rate controlled by the central bank, and E_t is the expectations operator. All variables are expressed in percentage deviations from their respective steady state values. The discount factor is denoted by $\beta < 1$, σ is the coefficient of relative risk aversion, and κ, the slope coefficient of the Phillips curve, depends negatively on the degree of price stickiness. Shocks to the Wicksellian natural real rate of interest are i.i.d. and are denoted by $r_t^n \sim \mathcal{N}(0,1)$. The precise origin of the shock plays no particular role for the subsequent analysis.

The central bank is uncertain about the slope coefficient κ_t. In particular, the policy-maker knows that his reference value $\bar{\kappa}$ might be subject to model distortions z to be explained below

$$\kappa_t = \bar{\kappa} + z_t \tag{3}$$

The central banker also faces an i.i.d. control error ξ_t with mean zero. Thus, policy is unable to use observations on inflation and the output gap to back out κ_t.

Monetary policy is unable to commit to the fully optimal policy plan. Instead, the central bank takes expectations as given and sets policy under discretion. The policy instrument, i.e. the short term interest rate, is set in order to minimize the welfare loss due to sticky-prices which is described in terms of inflation volatility, output gap volatility, and interest rate variance weighted by the parameters $\lambda_x, \lambda_i > 0$

$$\min_{\pi_t, x_t, i_t} \frac{1}{2} E_0 \sum_{t=0}^{\infty} \beta^t \left[(\pi_t - \pi^*)^2 + \lambda_x x_t^2 + \lambda_i {i_t}^2 \right] \tag{4}$$

where π^* is the constant inflation target. In the absence of misspecifications z_t, minimizing (4) subject to the model in (1) and (2) would give a set of first-order conditions, from which the optimal policy response to shocks could be computed.

The task is to reformulate the central bank's optimization problem such that the resulting policy rule performs well even if the model deviates from the reference model. We transform the minimization problem into a min-max problem. The central bank wants to minimize the maximum welfare loss due to model misspecifications by specifying an appropriate policy. To illustrate the problem, we introduce a fictitious second rational agent, the evil agent, whose only goal is to maximize the central bank's loss. The evil agent chooses a model from the available set of alternative models and the central bank chooses its policy optimally. Hence, the equilibrium is the outcome of a two-person game. Note that the evil agent is a convenient metaphor for the planner's cautionary behavior. Let z_t denote the evil agent's control variable, i.e. the parameter misspecification. The only constraint imposed upon the fictitious evil agent is his budget constraint requiring

$$E_t \sum_{\tau=0}^{\infty} \beta^{\tau} z_t^2 \leq \omega \tag{5}$$

Hence, the parameter ω measures the amount of misspecification the evil agent has available. The standard rational expectations solution for optimal monetary policy corresponds to $\omega = 0$, such that the evil agent's budget is empty.

3.2 The policy problem

Throughout the paper we assume that policy is unable to commit to the optimal inertial plan. Instead, policy is conducted under discretionary optimization. The policymaker solves

$$\min_{i_t} \max_{z_t} E_0 \sum_{t=0}^{\infty} \beta^t \left[(\pi_t - \pi^*)^2 + \lambda_x x_t^2 + \lambda_i {i_t}^2 \right] \tag{6}$$

subject to (1), (2), and (3). The Lagrangian of the policy problem can be written as follows

$$\begin{aligned} \min_{\pi_t, x_t, i_t} \max_{z_t} \mathcal{L} \quad = \quad & (\pi_t - \pi^*)^2 + \lambda_x x_t^2 + \lambda_i {i_t}^2 - \theta \left(z_t \right)^2 \qquad (7) \\ & - \mu_t^{\pi} \left(\pi_t - \beta E_t \pi_{t+1} - \left(\bar{\kappa} + z_t \right) x_t - \xi_t \right) \\ & - \mu_t^{x} \left(x_t - E_t x_{t+1} + \sigma^{-1} \left(i_t - E_t \pi_{t+1} - r_t^n \right) \right) \end{aligned}$$

where μ_t^{π} and μ_t^{x} denote the Lagrange multipliers associated to the inflation adjustment equation and the consumption Euler equation, respectively. The Lagrange parameter θ is inversely related to ω. Hence, the rational expectations case corresponds to $\theta \to \infty$.[7] A lower θ means that the central bank designs a policy which is appropriate for a wider set of possible misspecifications. Therefore, a lower θ is equivalent to a higher degree of robustness. The central bank plays a Nash game against the evil agent, who wants to maximize the welfare loss. Optimization under discretion results in the following

[7] In this case, the evil agent maximizes the welfare loss by choosing $z_t = 0$.

set of first-order conditions

$$\begin{aligned}
\lambda_x x_t + (\bar{\kappa} + z_t)\,\mu_t^{\pi} - \mu_t^{x} &= 0 \\
\pi_t - \pi^* - \mu_t^{\pi} &= 0 \\
\lambda_i i_t - \mu_t^{x}\sigma^{-1} &= 0 \\
-\theta z_t + \mu_t^{\pi} x_t &= 0
\end{aligned}$$

Together with the second condition, the fourth condition states that $z_t = (\pi_t - \pi^*)\, x_t \theta^{-1}$. The larger is the central bank's concern for robustness, i.e. the lower θ, the larger the model distortion. Likewise, the evil agent's choice of z_t positively depends on both the output gap and inflation. Hence, the worst case policy outcome against the central bank wishes to shield the economy is endogenous. Intuitively, model uncertainty matters most if inflation and output exhibit large deviations from their steady state values.[8]

The first order conditions can be combined to eliminate the Lagrange multipliers

$$\lambda_x x_t + \kappa t\,(\pi_t - \pi^*) - \sigma\lambda_i i_t = 0 \quad \text{with } \kappa_t = \bar{\kappa} + (\pi_t - \pi^*)\, x_t \theta^{-1} \tag{8}$$

When the inflation rate is above target and κ_t is known, the central bank has to raise the interest rate to contract the economy. When the central bank fears κ_t to be misspecified, a higher inflation rate also affects the slope coefficient κ_t. So not only does the central bank face an increase in inflation, but it also witnesses an increase in κ_t, i.e. its instrument becomes less effective in dampening aggregate demand. As a result, the size of the interest rate adjustment depends non-linearly on the inflation rate.

4 The optimal instrument rule

In this section we derive the optimal interest rate rule implied by the first order conditions.

4.1 Non-linear interest rate setting

Equation (8), which links all three endogenous variables, can be solved for i_t to obtain an expression that resembles a conventional Taylor rule augmented by a non-linear term

$$i_t = \frac{\bar{\kappa}}{\sigma\lambda_i}(\pi_t - \pi^*) + \frac{\lambda_x}{\sigma\lambda_i}x_t + \frac{1}{\theta\sigma\lambda_i}\left[x_t(\pi_t - \pi^*)^2\right] \tag{9}$$

[8]These first order conditions link the three endogenous variable irrespective of whether the misspecification of the underlying model actualy occurs, i.e. whether the reference model turns out to be undistorted.

The interest rate responds not only to the level of inflation and the output gap, but also to the product of the squared inflation deviation and the output gap. Note that the non-linear term disappears once we approach the rational expectations benchmark, i.e. $\theta \to \infty$. Suppose that the central bank observes an increase in inflation. Equation (10) shows that the interest rate response depends on the level of inflation and the output gap

$$\frac{\partial i_t}{\partial (\pi_t - \pi^*)} = \frac{\bar{\kappa}}{\sigma \lambda_i} + \frac{2}{\theta \sigma \lambda_i} \left[x_t (\pi_t - \pi^*) \right] \tag{10}$$

The interest rate response grows in the inflation rate. The higher the level of inflation, the more strongly (for a positive output gap) the central bank adjusts interest rates to fight an increase in inflation.[9] Furthermore, when the output gap is positive, the interest rate adjustment is stronger for positive inflation rates than for corresponding (in absolute terms) negative inflation rates. Hence, uncertainty not only introduces non-linearity, but also asymmetry into the optimal policy stance.

Likewise, the interest rate response to the output gap depends on the squared level of inflation

$$\frac{\partial i_t}{\partial x_t} = \frac{\lambda_x}{\sigma \lambda_i} + \frac{1}{\theta \sigma \lambda_i} (\pi_t - \pi^*)^2 \tag{11}$$

If inflation is high, the interest rate is raised more strongly to contract the economy than in a situation with moderate inflation. The precise interest rate step in this case depends on the parameterization.

4.2 Calibration

To visualize the degree of non-linearity in the Taylor rule, we choose standard parameter values to calculate the coefficients. In order to derive the interest rate rule, a positive interest weight in the central bank's loss function is essential. We choose to set $\lambda_x = 0.25$, which is a frequently used benchmark parameterization, and set the penalty on interest rate changes to $\lambda_i = 0.10$.

Choosing a parameter values for the robustness parameter θ is a critical issue. The drawback of the theory on robust control is that θ is a free parameter bounded only by zero. The rational expectations case corresponds to $\theta^{RE} = \infty$. We opt for a simplistic approach to determining a plausible robustness parameter. From the first-order conditions we know that $z_t = (\pi_t - \pi^*) x_t \theta^{-1}$. At the same time, the reference model specifies $\kappa_t = \bar{\kappa} + z_t$. It appears plausible to assume that the central bank considers only those misspecifications that feature a positively sloped Phillips curve. This means that $\bar{\kappa} + (\pi_t - \pi^*) x_t \theta^{-1} > 0$ must hold. Given the data used below, i.e.

[9] As in Giannoni (2002), the interest rate response to inflation within the Tayor rule increases as the central bank's degree of uncertainty becomes larger.

U.S. data on output gaps, inflation, and the inflation target from 1987 to 2004, this requires θ not to fall below 25. Therefore, we set $\theta^{robust} = 25$ to illustrate the effect of uncertainty in the calibration exercise. We assume an inflation target of zero, i.e. $\pi^* = 0$. The other parameters are set to $\bar{\kappa} = 0.10$, $\beta = 0.99$, and $\sigma = 1.80$. All of these values are standard in the literature.

The resulting interest rate response to inflation and output gap movements is depicted in figure (1). The non-linear response to inflation is clearly evident. A robustness-concerned central bank tolerates small fluctuations of inflation around the target, but forcefully counteracts larger deviations from target. Hence, the model also rationalizes that central banks frequently announce a target zone, typically $\pi^* \pm 1\%$, around their inflation target. Inflation is fought mildly inside the zone, but strongly once it leaves the target range.[10]

4.3 A note on endogenous weights

The period loss function of the form $L = (\pi_t - \pi^*)^2 + \lambda_x x_t^2 + \lambda_i {i_t}^2$ can be derived as an approximation to the households' utility function in the presence of transactions frictions that motivate a demand for money. Woodford (2003, p. 423-4) shows that the optimal weights λ_x and λ_i depend on the underlying model structure. In particular, they depend on κ perceived by the central bank

$$\lambda_x = \Omega_1 \kappa \quad \text{and} \quad \lambda_i = \Omega_2 \lambda_x \tag{12}$$

where Ω_1, $\Omega_2 > 0$ depend on the model parameters, including the interest rate semi-elasticity of money demand. This expression clearly shows the cross-equation restriction implied by the underlying theory. Any variation in κ should be reflected in variations of the weights λ_x and λ_i.[11] As a consequence, the misspecification z affects the weights the central bank attaches to conflicting objectives. If inflation increases, $\kappa = \bar{\kappa} + (\pi_t - \pi^*)\, x_t \theta^{-1}$ also increases for a positive output gap leading to larger weights λ_x and λ_i. This dampens the degree of non-linearity in (9).

5 Empirical Evidence

Is the non-linear instrument rule derived above empirically supported? To answer this question, we rewrite (9) in a form that corresponds to the large literature on estimated Taylor-type interest rate rules

[10] See Orphanides and Wieland (2000) for another model of inflation zone targeting.

[11] See Walsh (2005) for a detailed analysis of the consequences of endogenous weights for optimal monetary policy.

$$i_t = \bar{\imath} + \phi_\pi (\pi_t - \pi_t^*) + \phi_x x_t + \phi_{\pi^2 x} x_t (\pi_t - \pi_t^*)^2 + \varepsilon_t \tag{13}$$

where $\bar{\imath}$ is a constant and ϕ_i, ϕ_π, ϕ_x, and $\phi_{\pi^2 x}$ are reduced form coefficients to be estimated.[12] The inflation target, that is possibly time-varying, is denoted by π_t^*. We do not seek to estimate π_t^*. Instead, we take estimates of π_t^* from the literature on the estimation of the Federal Reserve's implicit inflation target as explained below.

In general, the empirical specification is kept as close as possible to the theoretical prediction. This, among other things, implies the absence of any form of interest rate dynamics. In the data, however, policy rates are extremely persistent processes. Although interest rate inertia is a stylized fact for almost every central bank, the literature has not yet reached a consensus as to the underlying determinants of interest rate persistence. If the period loss function penalizes the change in the policy instrument instead of the level, i.e. if the loss is given by $(\pi_t - \pi^*)^2 + \lambda_x x_t^2 + \lambda_i \Delta i_t{}^2$ (where $\Delta i_t = i_t - i_{t-1}$), the interest rate rule is

$$i_t = i_{t-1} + \frac{\bar{\kappa}}{\sigma \lambda_i} (\pi_t - \pi^*) + \frac{\lambda_x}{\sigma \lambda_i} x_t + \frac{1}{\theta \sigma \lambda_i} \left[x_t (\pi_t - \pi^*)^2 \right] \tag{14}$$

Thus, the lagged interest rate enters the policy rule, though including Δi_t in the loss function lacks a clear economic foundation. Nevertheless, we follow Cukierman and Muscatelli (2008) and accept a departure from the underlying model: We assume that only a fraction $1 - \phi_i$ of the current interest rate is related to contemporary inflation and output with the degree of interest rate inertia given by ϕ_i

$$i_t = (1 - \phi_i) \left[\bar{\imath} + \phi_\pi (\pi_t - \pi_t^*) + \phi_x x_t + \phi_{\pi^2 x} x_t (\pi_t - \pi_t^*)^2 \right] + \phi_i i_{t-1} + \varepsilon_t \tag{15}$$

Nevertheless, the baseline specification is one without interest rate inertia.[13] We also report results for a specification with $\phi_{\pi^2 x} = 0$, i.e. a model consistent with the absence of uncertainty, and with $\lambda_x = 0$ (which implies $\phi_x = 0$), i.e. for a "strict inflation targeting" regime.

These equations are estimated with least-squares using U.S. data for the period 1982:3-2004:1. The start of the sample period is given by the end of the Volcker disinflation, while the end of the sample is dictated by data availability. The inflation rate is the annualized rate of change of the personal consumption expenditure deflator (PCE) excluding food and energy prices. This is the Federal Reserve's preferred measure of inflation. The series of the inflation target is taken from Leigh (2008), who recovers the

[12] Clarida et al. (1998, 2000), Judd and Rudebusch (1998), and Jondeau et al. (2004) estimate similar, though linear, specifications.

[13] See Gerlach-Kristen (2004) for a recent analysis. The theoretical (non-inertial) policy rule can be interpreted as a "long-run" response.

unobservable inflation target based on a time-varying parameter model estimated with a Kalman filter. His results are the most recent estimates available in the literature (ending in 2004:1). The output gap is the deviation of (log) real GDP from the trend estimated by the Congressional Budget Office (CBO). It is well known that the GDP series that is available to the researcher now does not correspond to the data set that policy makers had at hand at each point in time. Data revisions often lead to large and persistent differences between real-time and revised data. Therefore, we also employ real-time estimates of the output gap (available until 2002:4) which were used by the staff of the Federal Reserve Board in preparing the Greenbook forecasts.[14] The inflation and output gap data is depicted in figure (2). The interest rate is the Federal Funds rate obtained from averaging monthly observations.

The results are presented in tables (1). Most importantly, the non-linear term $x_t\pi_t^2$ enters positively in all specifications. In line with the theory outlined above, the Fed has adjusted interest rate more aggressively the further inflation was away from steady state. Consider the baseline specification based on revised data for 1987:3-2004:1, i.e. after Greenspan took office as chairman. The interest rate response to inflation is 2.99 and to the output gap is 0.121. Both coefficients are fairly standard under the prevailing de-facto inflation targeting regime. Surprisingly, the coefficient on the non-linear term is large, $\phi_{\pi^2x} = 0.88$. Hence, uncertainty plays an important role in explaining interest rate setting.

To shed light on the role of parameter uncertainty over time we also estimate the model for different subsamples. In the longer sample that includes the early 1980s, the ϕ_{π^2x} coefficient is substantially smaller. Put differently, as the primary focus in the 1980s was to bring inflation back under control, uncertainty about the Phillips curve trade-off played a negligible (although still significant) role. Between 1994:1-2001:2, on the contrary, monetary policy faced a pervasive boom period with a persistently large positive output gap. In this period, the Fed was considering whether the output-inflation trade-off had changed due to favorable development in productivity. According to the model presented before, uncertainty about the Phillips curve translates into a large ϕ_{π^2x} coefficient in the estimated monetary policy rule. When estimated for this subsample, we indeed see an increase in this coefficient to 1.51. If we include interest rate inertia, the non-linear term remains significantly positive. Likewise, non-linearity remains important under strict inflation targeting.

As mentioned earlier, non-linearity in the Taylor rule does not only result from uncertainty of the policy maker about the true model of the economy, but also from non-

[14] Available under http://www.philadelphiafed.org/research-and-data/real-time-center/greenbook-data/gap-and-financial-data-set.cfm

linearity in the Phillips curve or from asymmetric central bank preferences. Dolado et al. (2005) show that a non-linear Phillips curve leads to on optimal policy rule that contains the product of the output gap and the inflation deviation from target. Surico (2007a) derives the optimal Taylor rule under asymmetric preferences, which features the squared inflation deviation and the squared output gap as separate arguments. How does the non-linear rule presented in this paper perform relative to these competing specifications? Table (2) presents the results of the Dolado et al. (2005) and the Surico (2007a) specification for two alternative output gap series. In accordance to Surico's finding, non-linearity stemming from asymmetric preferences does not matter in the post-1987 period. The coefficients on squared inflation and output are not significantly different from zero. Hence, asymmetric preferences cannot explain non-linearity in the policy role in the Greenspan-Bernanke era. Non-linearity arising from a non-standard Phillips curve, however, seems to matter as the coefficient $\phi_{\pi x}$ is significantly negative. Following the model of Dolado et al., this implies a concave Phillips curve. Figure (3) contrasts the squared residuals, i.e. the unexplained Federal Funds rate, obtained from this specification with those from our Taylor rule derived under uncertainty. It turns out that the model proposed in this paper leads to somewhat smaller residuals. Moreover, the empirical fit is remarkably better towards the end of the sample period. We therefore conclude that the Taylor rule derived under the assumption of uncertainty has explanatory power beyond those alternative non-linear specifications available in the literature. Taken together, the evidence presented in this section lends support to the notion that uncertainty about the Phillips curve slope is an important determinant of the observed interest rate setting behavior.

6 Conclusions

This paper showed that optimal monetary policy under parameter uncertainty can motivate a non-linear interest rate rule that is supported by U.S. data. While the linearity of the Phillips curve and the quadratic nature of the loss function are retained, the nonlinearity of the policy rule solely stems from the assumption of a min-max approach to parameter uncertainty. The crucial idea is that if the policymaker tries to avoid particularly bad outcomes, i.e. if she sets policy according to a min-max strategy, the maximum harm is endogenous and depends on the size of the output gap and the inflation rate. As a result, the policy response to inflation becomes stronger, the higher the inflation rate and the larger the output gap. The resulting non-linear Taylor rule is supported by U.S. data from the post-1982 period. In contrast to the bulk of the literature, these results do not stem from non-linearity in the Phillips curve or non-quadratic central preferences.

Certainly, the nature of parameter uncertainty analyzed here is overly simplistic. Not only is the central bank uncertain about a key parameter, but it also gains no information about this parameter over time even if the central bank repeatedly plays against the evil agent. However, the basic principle appears to be relevant to interpret actual policy decisions.

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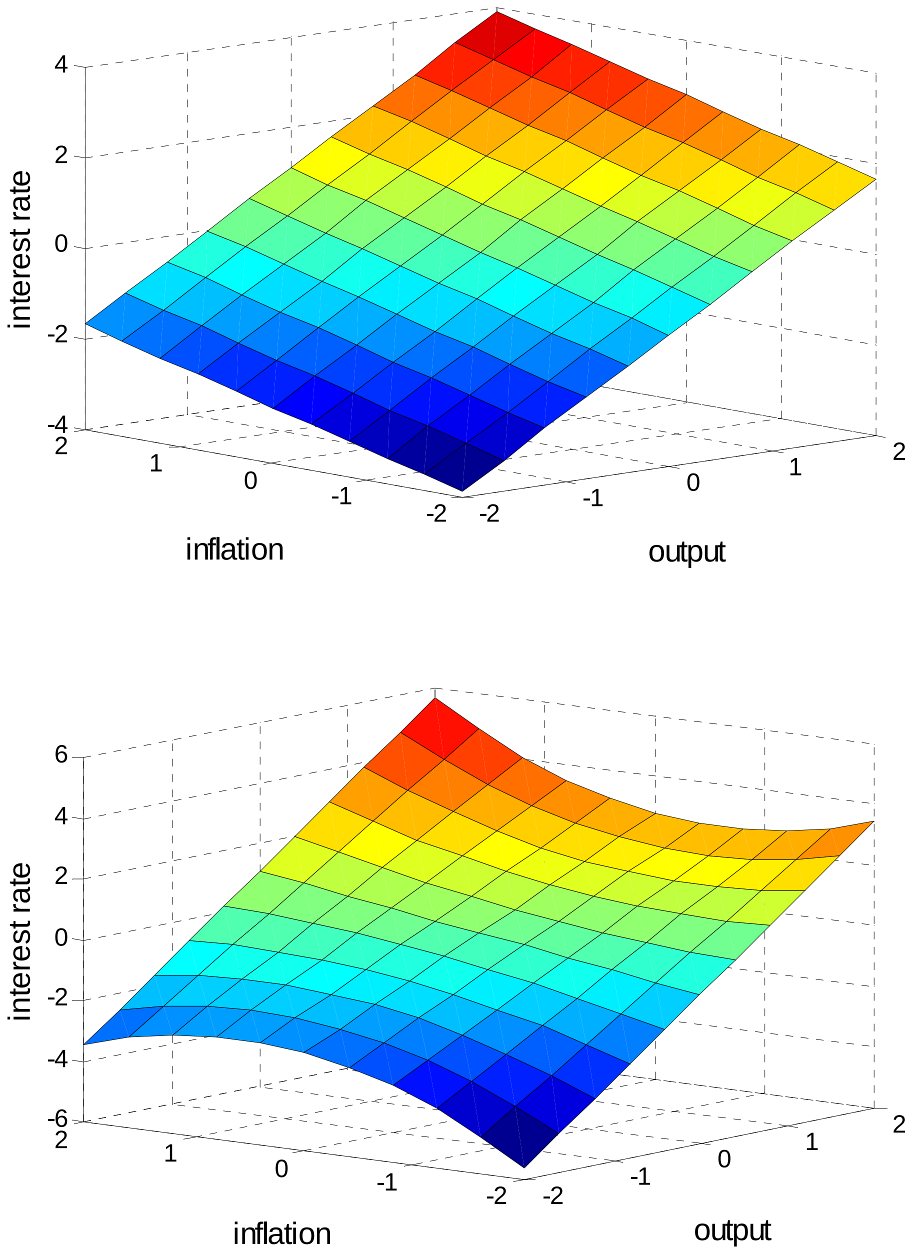

Figure 1: The interest rate as described by a linear (upper panel) policy rule and a non-linear (lower panel) policy rule

Table 1: Estimates of non-linear Taylor rules based on parameter uncertainty

output gap series	sample	Parameter estimates				R^2
		ϕ_π	ϕ_x	$\phi_{\pi^2 x}$	ϕ_i	
revised	1982:3 - 2004:1	3.456 (0.239)***	0.336 (0.089)***	0.061 (0.016)***		0.77
		2.914 (0.277)***	0.455 (0.105)***			0.73
		3.300 (0.342)***		0.090 (0.017)***		0.72
		4.503 (0.534)***	0.510 (0.5125)***	0.162 (0.039)***	0.765 (0.047)***	0.97
		3.091 (0.792)***	0.925 (0.187)***		0.803 (0.044)***	0.95
revised	1987:3-2004:1	2.993 (0.456)***	0.215 (0.102)**	0.883 (0.442)**		0.68
		3.333 (0.418)***	0.352 (0.099)***			0.66
		2.785 (0.432)***		1.430 (0.408)***		0.67
revised	1994:1-2001:2	1.932 (0.545)***	0.252 (0.090)***	1.514 (0.659)**		0.40
		1.096 (0.557)*	0.295 (0.134)**			0.25
		1.367 (0.673)*		1.848 (0.858)**		0.20
real-time	1987:3-2002:4	2.248 (0.530)***	0.177 (0.095)*	1.734 (0.492)***		0.61
		2.735 (0.545)***	0.333 (0.098)***			0.59
		2.136 (0.494)***		2.402 (0.448)***		0.60

Notes: Results from least-squares estimation. Newey-West corrected standard errors in parenthesis. A significance level of 1%, 5%, and 10% is indicated by ***, **, and *.

Table 2: Estimates of alternative non-linear Taylor rules

output gap series	sample	Parameter estimates					R^2
		ϕ_π	ϕ_x	$\phi_{\pi x}$	ϕ_{π^2}	ϕ_{x^2}	
Optimal Taylor rule based on non-linear Phillips curve, i.e. Dolado et al. (2005) $i_t = \phi_\pi (\pi_t - \pi_t^*) + \phi_x x_t + \phi_{\pi x} x_t (\pi_t - \pi_t^*) + \varepsilon_t$							
revised	1987:3-2004:1	3.137 $(0.470)^{***}$	0.218 $(0.109)^{*}$	-0.565 $(0.297)^{*}$			0.70
real-time	1987:3-2002:4	2.739 $(0.506)^{***}$	0.184 $(0.109)^{*}$	-0.801 $(0.359)^{**}$			0.62
Optimal Taylor rule based on asymmetric preferences, i.e. Surico (2007a) $i_t = \phi_\pi (\pi_t - \pi_t^*) + \phi_x x_t + \phi_{x^2} x_t^2 + \phi_{\pi^2} (\pi_t - \pi_t^*)^2 + \varepsilon_t$							
revised	1987:3-2004:1	3.426 $(0.389)^{***}$	0.309 $(0.113)^{***}$		0.038 (0.059)	-0.384 (0.683)	0.68
real-time	1987:3-2002:4	2.850 $(0.606)^{***}$	0.367 $(0.110)^{***}$		0.027 (0.056)	-0.250 (1.150)	0.59

Notes: Results from least-squares estimation. Newey-West corrected standard errors in parenthesis. A significance level of 1%, 5%, and 10% is indicated by ***, **, and *.

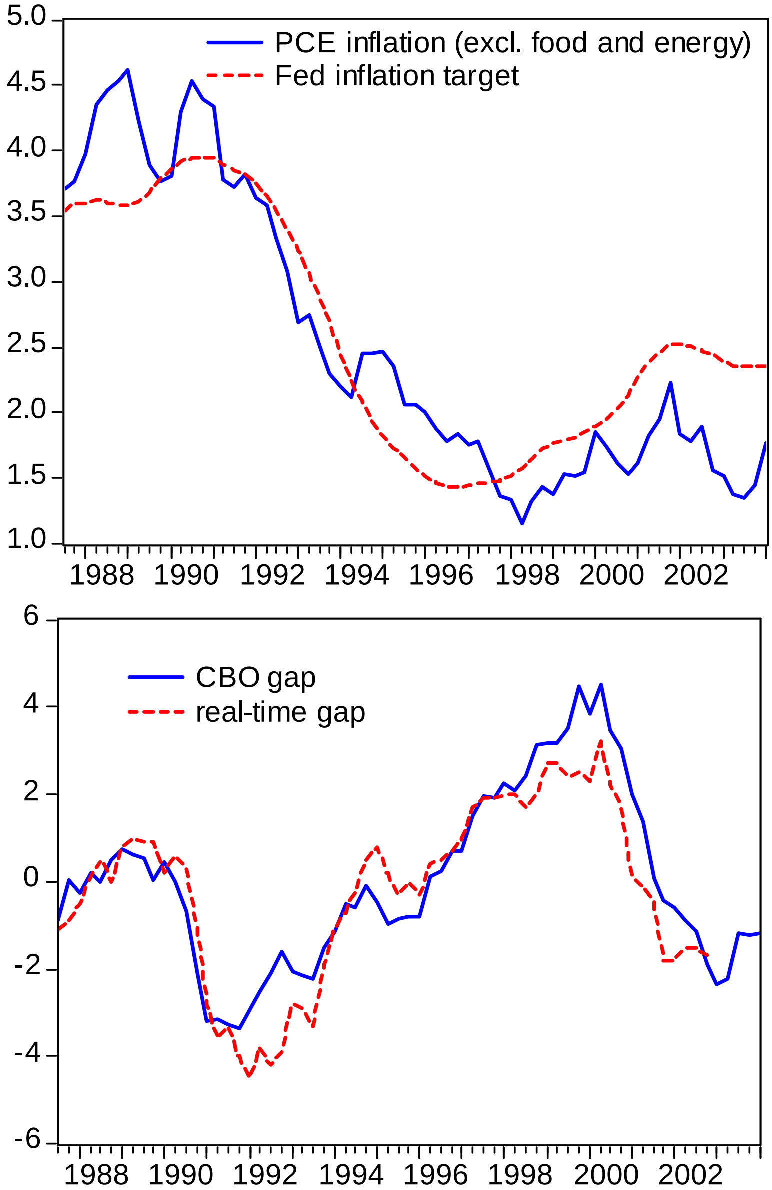

Figure 2: Output and inflation data (sources: see main text)

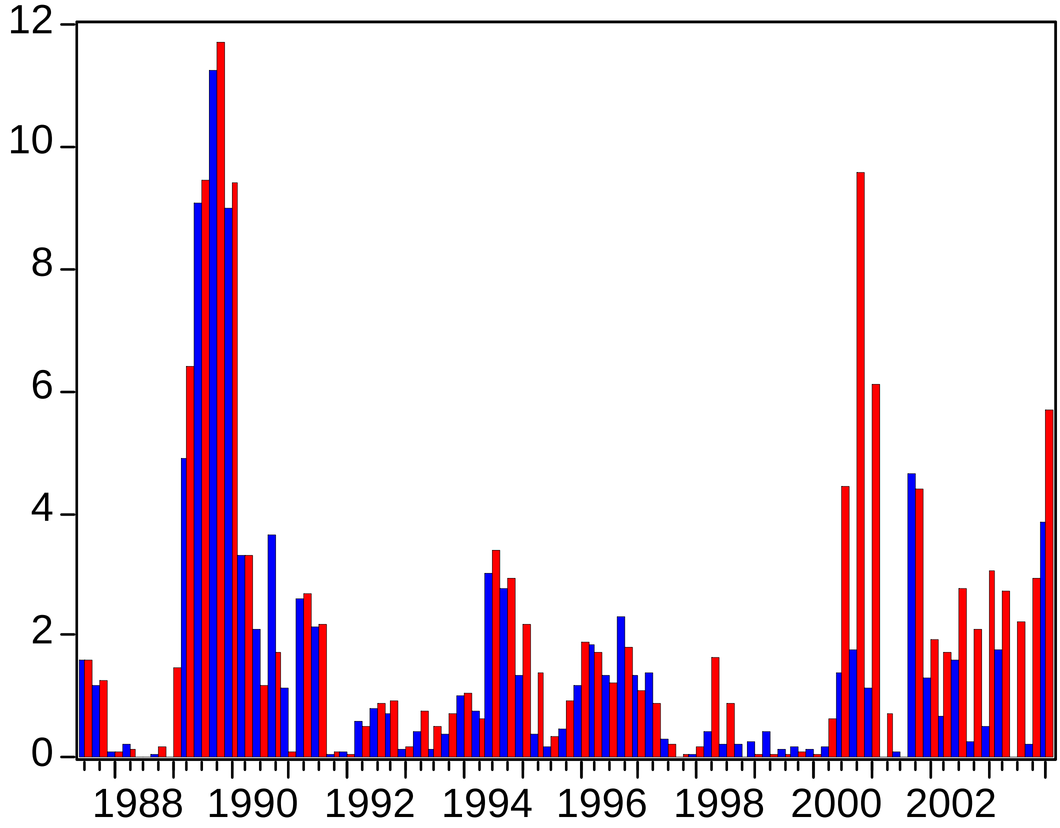

Figure 3: Unexplained Federal Funds rate in this paper's non-linear robust rule (blue bars) and the rule based on non-linearity in the Phillips curve following Dolado et al. (red bars)